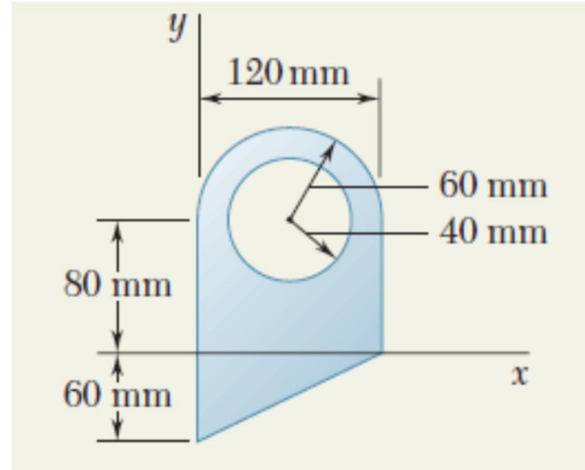
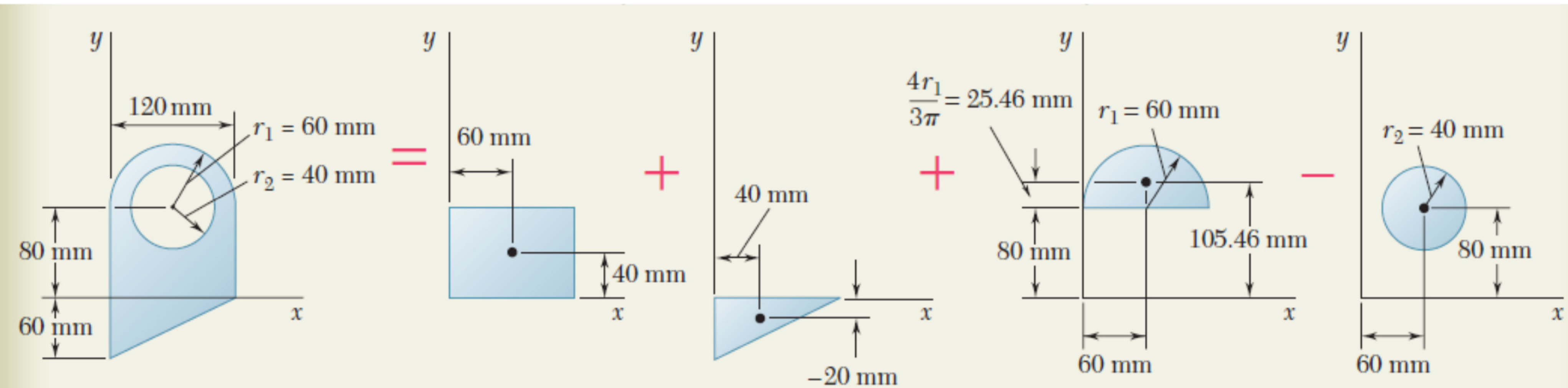
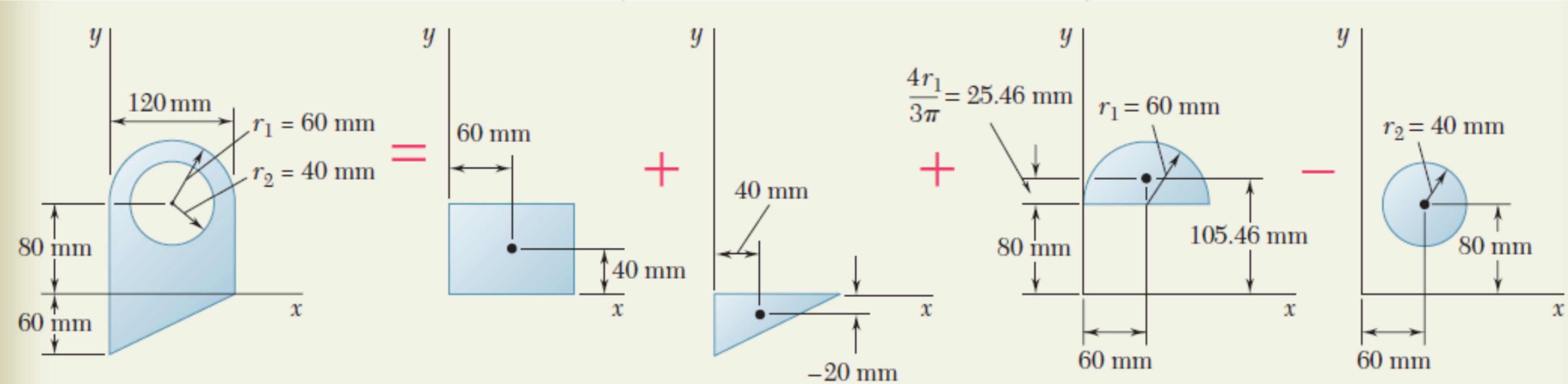
For the plane area shown, determine (*a*) the first moments with respect to the x and y axes, (*b*) the location of the centroid.







Component	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm^3	$\bar{y}A$, mm^3
Rectangle	$(120)(80) = 9.6 \times 10^3$	60	40	$+576 \times 10^3$	$+384 \times 10^3$
Triangle	$\frac{1}{2}(120)(60) = 3.6 \times 10^3$	40	-20	$+144 \times 10^3$	-72×10^3
Semicircle	$\frac{1}{2}\pi(60)^2 = 5.655 \times 10^3$	60	105.46	$+339.3 \times 10^3$	$+596.4 \times 10^3$
Circle	$-\pi(40)^2 = -5.027 \times 10^3$	60	80	-301.6×10^3	-402.2×10^3
	$\Sigma A = 13.828 \times 10^3$			$\Sigma\bar{x}A = +757.7 \times 10^3$	$\Sigma\bar{y}A = +506.2 \times 10^3$

First Moments of the Area.

$$Q_x = \Sigma\bar{y}A = 506.2 \times 10^3 \text{ mm}^3$$
$$Q_y = \Sigma\bar{x}A = 757.7 \times 10^3 \text{ mm}^3$$

Location of Centroid.

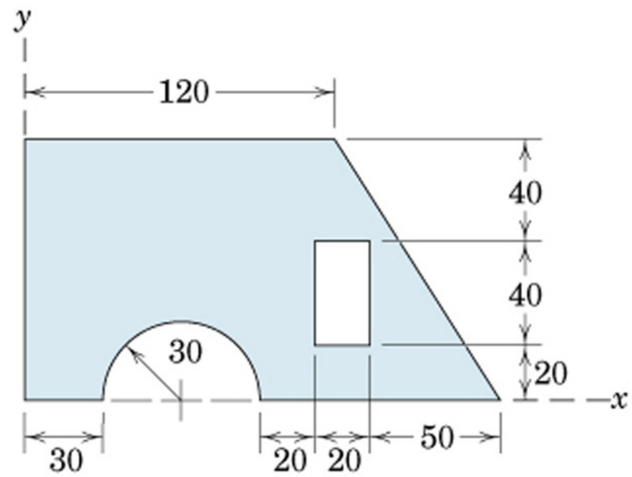
$$\bar{X}\Sigma A = \Sigma\bar{x}A: \qquad \bar{X}(13.828 \times 10^3 \text{ mm}^2) = 757.7 \times 10^3 \text{ mm}^3$$
$$\bar{X} = 54.8 \text{ mm}$$

$$\bar{Y}\Sigma A = \Sigma\bar{y}A: \qquad \bar{Y}(13.828 \times 10^3 \text{ mm}^2) = 506.2 \times 10^3 \text{ mm}^3$$
$$\bar{Y} = 36.6 \text{ mm}$$

Center of Mass and Centroids: Composite Bodies and Figures

Example:
Locate the centroid of the shaded area

Solution: Divide the area into four elementary shapes: Total Area = $A_1 + A_2 - A_3 - A_4$

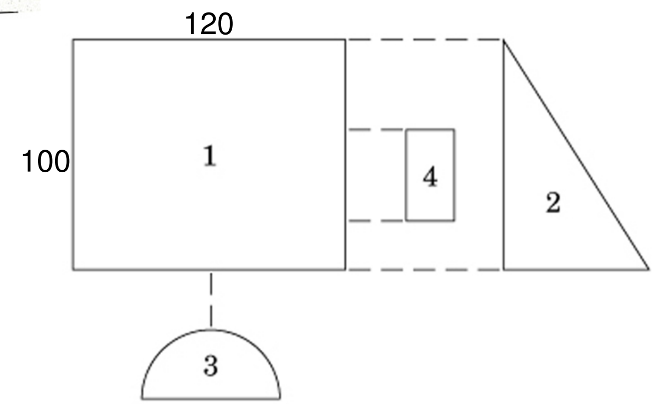


Dimensions in millimeters

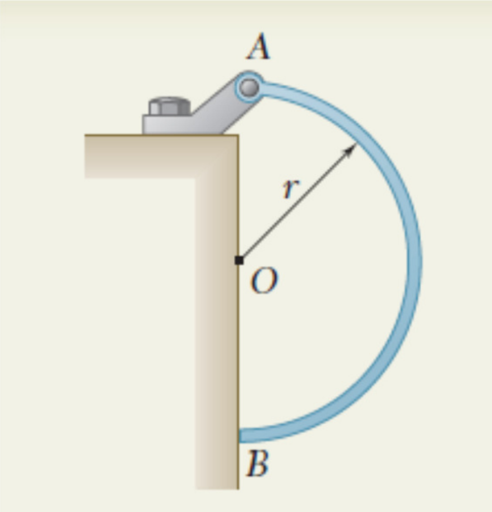
PART	A mm^2	$\bar{x}$ mm	$\bar{y}$ mm	$\bar{x}A$ mm^3	$\bar{y}A$ mm^3
1	12 000	60	50	720 000	600 000
2	3000	140	100/3	420 000	100 000
3	−1414	60	12.73	−84 800	−18 000
4	−800	120	40	−96 000	−32 000
TOTALS	12 790			959 000	650 000

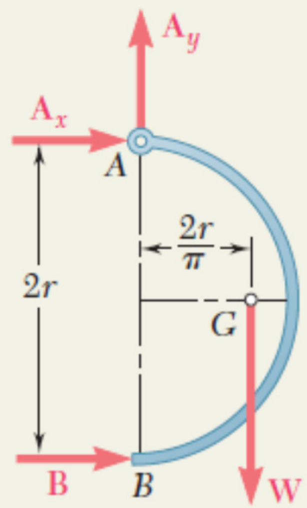
$$\left[\bar{X} = \frac{\Sigma A\bar{x}}{\Sigma A}\right] \qquad \bar{X} = \frac{959\ 000}{12\ 790} = 75.0 \text{ mm}$$

$$\left[\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A}\right] \qquad \bar{Y} = \frac{650\ 000}{12\ 790} = 50.8 \text{ mm}$$



A uniform semicircular rod of weight W and radius r is attached to a pin at A and rests against a frictionless surface at B. Determine the reactions at A and B.





$$+\circlearrowleft \Sigma M_A = 0: \quad B(2r) - W\left(\frac{2r}{\pi}\right) = 0$$

$$B = +\frac{W}{\pi}$$

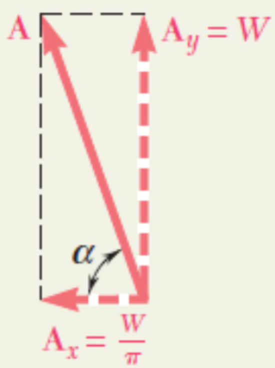
$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x + B = 0$$

$$A_x = -B = -\frac{W}{\pi} \qquad \mathbf{A}_x = \frac{W}{\pi} \leftarrow$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - W = 0 \qquad \mathbf{A}_y = W \uparrow$$

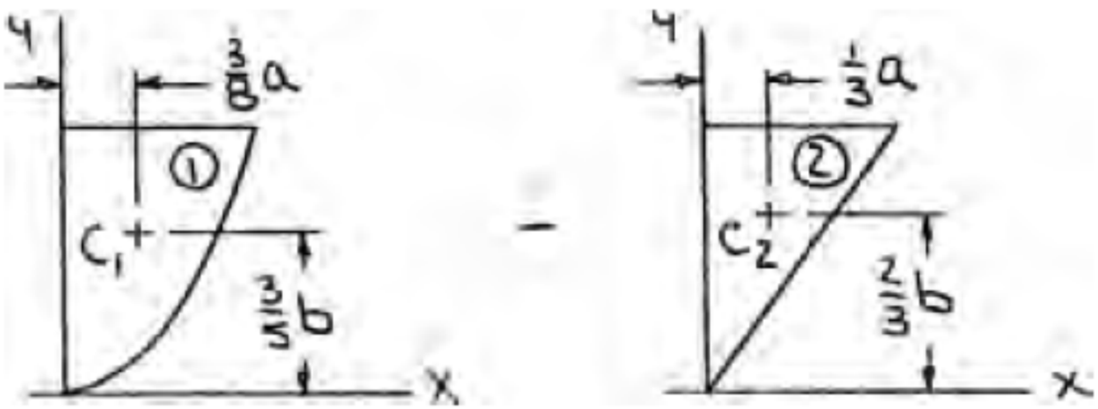
$$A = \left[W^2 + \left(\frac{W}{\pi}\right)^2\right]^{1/2}$$

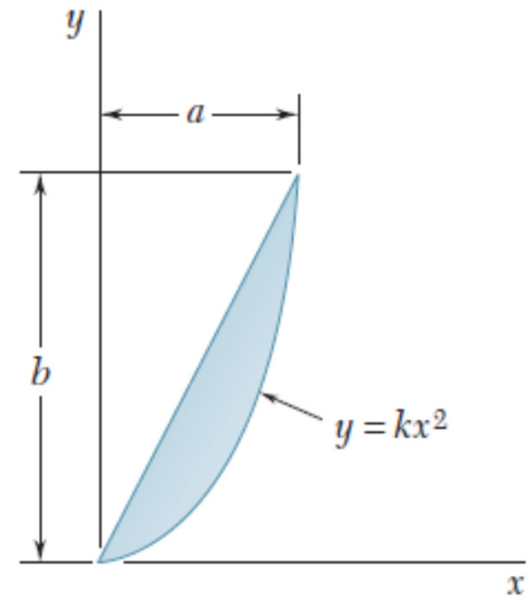
$$\tan\alpha = \frac{W}{W/\pi} = \pi$$



$$\mathbf{A} = 1.049W \measuredangle 72.3^\circ \qquad \mathbf{B} = 0.318W \rightarrow$$

For the area shown, determine the ratio a/b for which $\bar{x} = \bar{y}$





	A	$\bar{x}$	$\bar{y}$	$\bar{x}A$	$\bar{y}A$
1	$\frac{2}{3}ab$	$\frac{3}{8}a$	$\frac{3}{5}b$	$\frac{a^2b}{4}$	$\frac{2ab^2}{5}$
2	$-\frac{1}{2}ab$	$\frac{1}{3}a$	$\frac{2}{3}b$	$-\frac{a^2b}{6}$	$-\frac{ab^2}{3}$
Σ	$\frac{1}{6}ab$			$\frac{a^2b}{12}$	$\frac{ab^2}{15}$

Then $$\bar{X}\,\Sigma A = \Sigma\,\bar{x} A$$

$$\bar{X}\left(\frac{1}{6}ab\right) = \frac{a^2 b}{12}$$

or $$\bar{X} = \frac{1}{2}a$$

$$\bar{Y}\,\Sigma\,A = \Sigma\,\bar{y}\,A$$

$$\bar{Y}\left(\frac{1}{6}ab\right) = \frac{ab^2}{15}$$

or $$\bar{Y} = \frac{2}{5}b$$

Now $$\bar{X} = \bar{Y} \Rightarrow \quad \frac{1}{2}a = \frac{2}{5}b \qquad \text{or} \quad \frac{a}{b} = \frac{4}{5} \blacktriangleleft$$